

Intersecting Noncommutative D-branes and Baryons in Magnetic Fields

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ABSTRACT

We study supersymmetric intersecting configurations of D-branes with B -field backgrounds. Noncommutative D-brane or M-brane pairs can intersect supersymmetrically over $(p-1)$ -brane, as well as over $(p-2)$ -brane like ordinary branes. $d=10$ and $d=11$ supergravity solutions are obtained and the supersymmetry projection rule is examined. As an application we study a noncommutative D7-brane probe in noncommutative D3-brane background, intersecting at noncommutative plane, which describes BPS baryons of noncommutative gauge theory in the context of AdS/CFT correspondence.

1 Introduction

The relevance of noncommutative geometry to superstring theory was first noted by Connes, Douglas and Schwarz [1], who showed that the compactification of Matrix theory gives Yang-Mills theory defined on noncommutative torus when a constant background three-form field potential is turned on. Recently Seiberg and Witten [2] showed that Born-Infeld theory on ordinary space with nonzero gauge background and noncommutative Born-Infeld theory are equivalent up to field redefinition, and by adopting different regularization schemes one can derive both theories from the dynamics of open strings with mixed boundary conditions.

Following this one of the most important subject is the study of supergravity solutions of D-branes with Neveu-Schwarz-Neveu-Schwarz (NS-NS) B -fields, or when we generalize to M-theory, M-branes with three-form C -field potential backgrounds. It is now well-established that large- N supersymmetric Yang-Mills theories are holographic descriptions of string theories in N D-brane backgrounds [3]. Various supergravity solutions which are dual to noncommutative Yang-Mills theories are presented in [4, 5]. They are characterized by a nonzero B on the boundary but reduce to the usual Dp -brane solutions at the horizon. In this paper we will mainly consider only one magnetic component of B which is nonzero, and then from the Ramond-Ramond (RR) fields we can see that the supergravity solutions describe bound states of Dp and $D(p-2)$ branes. From now on we will call these 1/2 BPS D-branes with nontrivial B noncommutative D-branes.

The next most essential objects in string theory which are relevant to noncommutative geometry should be intersecting noncommutative branes with nontrivial B . Supergravity solutions for such configurations are obtained in [6] using the $SL(2, Z)$ electro-magnetic duality of M-theory compactifications. For ordinary brane pairs without B -fields the necessary condition for supersymmetry is that the number of overall transverse directions must be a multiple of 4. In particular a Dp -brane pair can make BPS intersecting configuration over $D(p-2)$ -brane. In this paper we point out that with nonvanishing B , a Dp -brane pair can intersect over $(p-1)$ dimensional space, preserving 1/4 of the supersymmetries. An important consequence of this reasoning applies to the study of baryonic branes in the AdS/CFT correspondence. In N D3-brane background, a D5-brane plays a role of the source for N fundamental strings which constitute a baryon in the dual theory. We can study the baryons in terms of a soliton of D5-brane Born-Infeld action, when the D5-brane is wrapped on S^5 part of $AdS_5 \times S^5$. Now about the noncommutative version of AdS/CFT correspondence, it turns out that a single D5-brane in noncommutative D3-brane background cannot enjoy BPS configuration. But we will see that a D7-brane with appropriate Born-Infeld gauge field excitations can. The baryonic D5-brane wrapping S^5 must be

modified to a D7-brane which also wraps the S^5 and extends on the noncommutative part of the D3 worldvolume.

The plan of this paper is as follows. In Section 2 starting with the supergravity solution for noncommutative branes we present various BPS intersecting noncommutative brane configurations. Section 3 discusses the supersymmetries of intersecting noncommutative branes using both the supergravity solution and the κ -symmetry of D-brane probe embedded in the supergravity backgrounds. In Section 4 we find the equation governing the shape of baryonic noncommutative D7-brane from the Dirac-Born-Infeld action. The last section provides a brief discussion.

2 Supergravity Solutions

The supergravity solution of a D3-brane in a constant NS-NS B field background is obtained in [4, 5].

$$\begin{aligned} ds_{\text{string}}^2 &= f^{-1/2}[-dx_0^2 + dx_1^2 + h(dx_2^2 + dx_3^2)] + f^{1/2}(dr^2 + r^2 d\Omega_5^2), \\ f &= 1 + \frac{\alpha'^2 R^4}{r^4}, \quad h^{-1} = \sin^2 \varphi f^{-1} + \cos^2 \varphi, \\ B_{23}^{\text{NS}} &= \tan \varphi f^{-1} h, \quad e^{2\phi} = g^2 h, \\ F_{01r}^{\text{RR}} &= \frac{1}{g} \sin \varphi \partial_r f^{-1}, \quad F_{0123r}^{\text{RR}} = \frac{1}{g} \cos \varphi h \partial_r f^{-1}. \end{aligned} \tag{1}$$

Note that $B_{23} = 0$ close to the horizon, while $B_{23} = \tan \varphi$ on the boundary. From the RR gauge fields it is evident that this solution has a D-string charge as well as a D3-brane. Technically there are two ways to get above solution [4, 7]. We can either (1) start with a D1-brane solution with a constant B_{23}^{NS} and delocalized in 23-directions, and then T-dualize twice, along x_2 and x_3 , or (2) consider a D2-brane, extended along 12 and delocalized in x_3 -direction, rotate the solution in 23-plane and then T-dualize along x_3 . These two methods produce gauge equivalent solutions with different values of B_{23}^{NS} . Method (1) gives above solution and method (2) gives the same solution with $B_{23}^{(2)} = B_{23}^{(1)} - \tan \varphi$, which means we exchange two solutions at the expense of turning on a U(1) Born-Infeld field strength on the D3-brane worldvolume. In the context of holography r/α' represents energy scale of the boundary theory. The fact that $B^{\text{NS}} = 0$ at $r = 0$ is consistent with the classical expectation that the noncommutativity has no effect on IR behaviour. We should comment here that this is usually not true in quantum field theories defined on noncommutative spaces. It turns out that in loop integrals the high momentum modes can generate long range forces and there is a mixing of IR and UV physics in noncommutative field theory [8, 9].

To decouple the asymptotic region and relate to noncommutative Yang-Mills theory, we rescale the parameters in the following way [5, 10],

$$\begin{aligned}\alpha' &\rightarrow 0, & \tan \varphi &= \frac{\Theta}{\alpha'}, \\ x_{0,1} &= \bar{x}_{0,1}, & x_{2,3} &= \frac{\alpha'}{\Theta} \bar{x}_{2,3}, \\ r &= \alpha' u, & g &= \alpha' \bar{g},\end{aligned}\tag{2}$$

where $\Theta, u, \bar{g}, \bar{x}_i$ are fixed. Especially Θ is the noncommutativity parameter, $[\bar{x}_2, \bar{x}_3] \sim \Theta$. Several basic aspects of noncommutative AdS/CFT correspondence, like the calculation of correlation functions and Wilson loops, were studied using the above supergravity solution and decoupling limit [5, 10].

Now we turn to intersecting D-brane solutions with nontrivial B -fields. In [6] the solutions were introduced as non-threshold bound states of orthogonally intersecting branes while in this paper we rather call them intersecting noncommutative D-branes. We use here the method (2) explained above, i.e. T-dualizing D-branes at an angle.

Let us start with a D4-brane pair intersecting over a D2-brane with B fields. To obtain the supergravity solution we first write a Type IIB solution describing a D3-brane pair intersecting at a D1-brane, which can be written easily using the harmonic superposition rule,

$$\begin{aligned}ds_{\text{string}}^2 &= f_1^{-1/2} f_2^{-1/2} (-dx_0^2 + dx_3^2) + f_1^{-1/2} f_2^{1/2} (dx_1^2 + dx_2^2) \\ &\quad + f_1^{1/2} f_2^{-1/2} (dx_5^2 + dx_6^2) + f_1^{1/2} f_2^{1/2} (dx_4^2 + dr^2 + r^2 d\Omega_2^2), \\ F_{0123r} &= \frac{1}{g} \partial_r f_1^{-1}, & F_{0356r} &= \frac{1}{g} \partial_r f_2^{-1}, & f_{1,2} &= 1 + \frac{\alpha'^{1/2} R_{1,2}}{r}.\end{aligned}\tag{3}$$

Note that we prepared $D3(123) \perp D3(356)$ delocalized in x^4 . We rotate above solution by introducing new coordinates,

$$\begin{aligned}x_3 &= \tilde{x}_3 \cos \varphi - \tilde{x}_4 \sin \varphi, \\ x_4 &= \tilde{x}_3 \sin \varphi + \tilde{x}_4 \cos \varphi.\end{aligned}$$

And we T-dualize along \tilde{x}_4 , using the T-duality relation between supergravity solutions with RR fields [11] to get the following solution,

$$\begin{aligned}ds_{\text{string}}^2 &= f_1^{-1/2} f_2^{-1/2} [-dx_0^2 + h(dx_3^2 + dx_4^2)] + f_1^{1/2} f_2^{-1/2} (dx_1^2 + dx_2^2) \\ &\quad + f_1^{-1/2} f_2^{1/2} (dx_5^2 + dx_6^2) + f_1^{1/2} f_2^{1/2} (dr^2 + r^2 d\Omega_2^2), \\ e^{2\phi} &= g^2 f_1^{-1/2} f_2^{-1/2} h, & h^{-1} &= \cos^2 \varphi + f_1^{-1} f_2^{-1} \sin^2 \varphi, \\ B_{34} &= f_1^{-1} f_2^{-1} h \tan \varphi, \\ F &= \frac{\cos \varphi}{g} R_1 dx_5 \wedge dx_6 \wedge \epsilon_2 + \frac{\cos \varphi}{g} R_2 dx_1 \wedge dx_2 \wedge \epsilon_2 \\ &\quad + \frac{\sin \varphi}{g} df_1^{-1} dx_0 \wedge dx_1 \wedge dx_2 + \frac{\sin \varphi}{g} df_2^{-1} dx_0 \wedge dx_5 \wedge dx_6,\end{aligned}\tag{4}$$

where ϵ_2 is the volume element of a unit radius 2-sphere. Note that we shifted B_{34} by $\tan \varphi$ to make it vanish at infinity and $B_{34} = \tan \varphi$ at $r = 0$. In the next section it will be shown that this shift guarantees the supersymmetry of a D4(1234) and a D4(3456) probe *without* Born-Infeld U(1) gauge field in the above background. By varying φ we note that at $\varphi = 0$ we have a D4-brane pair of (1234) and (3456), while at $\varphi = \pi/2$ we have a D2-brane pair intersecting over a point, extended on (12) and (34) plane respectively.

Now let us consider a noncommutative D3-brane pair intersecting over a 2d plane. We start with an intersecting D2-brane pair $D2(12) \perp D2(34)$,

$$\begin{aligned} ds_{\text{string}}^2 &= -f_1^{-1/2} f_2^{-1/2} dx_0^2 + f_1^{-1/2} f_2^{1/2} (dx_1^2 + dx_2^2) \\ &\quad + f_1^{1/2} f_2^{-1/2} (dx_3^2 + dx_4^2) + f_1^{1/2} f_2^{1/2} (dr^2 + r^2 d\Omega_4^2), \\ f_{1,2} &= 1 + \frac{\alpha'^{3/2} R_{1,2}^3}{r^3}, \quad e^{2\phi} = g^2 f_1^{1/2} f_2^{1/2}, \\ F_{012r} &= \frac{1}{g} \partial_r f_1^{-1}, \quad F_{034r} = \frac{1}{g} \partial_r f_2^{-1}. \end{aligned} \quad (5)$$

Then rotate the solution in 23-directions like we did above, and perform T-duality along \tilde{x}_3 , to get

$$\begin{aligned} ds_{\text{string}}^2 &= -f_1^{-1/2} f_2^{-1/2} dx_0^2 + f_1^{-1/2} f_2^{1/2} dx_1^2 + f_1^{-1/2} f_2^{-1/2} k (dx_2^2 + dx_3^2), \\ &\quad + f_1^{1/2} f_2^{-1/2} dx_4^2 + f_1^{1/2} f_2^{1/2} (dr^2 + r^2 d\Omega_4^2), \\ e^{2\phi} &= g^2 k, \quad k^{-1} = f_1^{-1} \cos^2 \varphi + f_2^{-1} \sin^2 \varphi, \\ B_{23} &= -\tan \varphi f_1^{-1} k, \\ F_{01r} &= \frac{1}{g} \cos \varphi \partial_r f_1^{-1}, \quad F_{04r} = -\frac{1}{g} \sin \varphi \partial_r f_2^{-1}, \\ F_{0123r} &= \frac{1}{g} \sin \varphi k f_2^{-1} \partial_r f_1^{-1}, \quad F_{0234r} = \frac{1}{g} \cos \varphi k f_1^{-1} \partial_r f_2^{-1}. \end{aligned} \quad (6)$$

At $\varphi = 0$ or $\varphi = \pi/2$ we see that this system reduces to D3-brane and D1-brane intersecting at a point, but at generic values we have a pair of D3-branes with non-trivial B -fields. We also shifted the value of B by $\tan \varphi$ to make $B = \tan \varphi$ on the boundary, but differently from previous examples it is nonzero at the horizon. In this background D3-brane probes extended along (123) and (234)-directions cannot be supersymmetric with the same values of $\mathcal{F} = F - B$. In the above background, a D3-brane on (123)-space is supersymmetric with $\mathcal{F}_{23} = -\cot \varphi f_1^{-1} k$, while a D3-brane on (234)-space becomes supersymmetric with $\mathcal{F}_{23} = \tan \varphi f_2^{-1} k$. This will be checked in the next section using the κ -supersymmetry of D-brane action.

Like ordinary intersecting brane solutions without B -fields, we can obtain other solutions via T-duality. For example if we T-dualize above solution along x_5 , we get a

noncommutative D4-brane pair intersecting over 3-brane, i.e. $D4(1235) \perp D4(2345)$, with B_{23} .

M-theory generalization is also easily achieved using the relation between IIA string theory and M-theory, which can be found for example in [11]. In this process both NS-NS 2-form B -field and RR 3-form field are united into M-theory 3-form field potential C . A noncommutative M5-brane solution $M5(12345)$ with C_{012} and C_{345} , which can be considered as a bound state of $M5(12345)$ and $M2(12)$ can be written easily from the noncommutative D4-brane on (1234) -space with B_{34} ,

$$\begin{aligned} ds_{11}^2 &= f^{-1/3} h^{-1/3} [-dx_0^2 + dx_1^2 + dx_2^2 + h(dx_3^2 + dx_4^2 + dx_5^2) + f(dr^2 + r^2 d\Omega_4^2)], \\ f &= 1 + \frac{R^3}{r^3}, \quad h^{-1} = f^{-1} \sin^2 \varphi + \cos^2 \varphi, \\ dC_3 &= \sin \varphi df^{-1} \wedge dx_0 \wedge dx_1 \wedge dx_2 - 6 \tan \varphi d(f^{-1}h) \wedge dx_3 \wedge dx_4 \wedge dx_5 \\ &\quad + \cos \varphi 3R^3 \epsilon_4, \end{aligned} \tag{7}$$

where ϵ_5 is the volume element of a unit radius 5-sphere.

For a noncommutative M5-brane pair intersecting over 3d space, $M5(12345) \perp M5(34567)$ with $C_{345}, C_{012}, C_{067}$, the supergravity solution can be obtained from the IIA solution eq.(4),

$$\begin{aligned} ds^2 &= f_1^{-1/3} f_2^{-1/3} h^{-1/3} [-dx_0^2 + f_2(dx_1^2 + dx_2^2) + h(dx_3^2 + dx_4^2 + dx_5^2) \\ &\quad + f_1(dx_6^2 + dx_7^2) + f_1 f_2(dr^2 + r^2 d\Omega_2^2)], \\ f_{1,2} &= 1 + \frac{R_{1,2}}{r}, \quad h^{-1} = \cos^2 \varphi + f_1^{-1} f_2^{-1} \sin^2 \varphi, \\ dC_3 &= \sin \varphi df_1^{-1} \wedge dx_0 \wedge dx_1 \wedge dx_2 + \sin \varphi df_2^{-1} \wedge dx_0 \wedge dx_6 \wedge dx_7 \\ &\quad + R_1 \cos \varphi dx_6 \wedge dx_7 \wedge \epsilon_2 + R_2 \cos \varphi dx_1 \wedge dx_2 \wedge \epsilon_2 \\ &\quad - 6 \tan \varphi d(f_1^{-1} f_2^{-1} h) \wedge dx_3 \wedge dx_4 \wedge dx_5. \end{aligned} \tag{8}$$

A noncommutative M5-brane pair can also intersect over 4d space, $M5(12346) \perp M5(23456)$ with $C_{234}, C_{015}, C_{016}$. The D=11 supergravity condition can be obtained by lifting eq.(6) twice,

$$\begin{aligned} ds^2 &= k^{-1/3} f_1^{-1/3} f^{-1/3} [(-dx_0^2 + dx_1^2) + k(dx_2^2 + dx_3^2 + dx_4^2) \\ &\quad + f_2^{-1} dx_5^2 + f_1^{-1} dx_6^2 + f_1 f_2(dr^2 + r^2 d\Omega_3^2)], \\ f_{1,2} &= 1 + \frac{R_{1,2}^2}{r^2}, \quad k^{-1} = f_1^{-1} \cos^2 \varphi + f_2^{-1} \sin^2 \varphi, \\ dC_3 &= \cos \varphi df_1^{-1} \wedge dx_0 \wedge dx_1 \wedge dx_5 - \sin \varphi df_2^{-1} \wedge dx_0 \wedge dx_5 \wedge dx_6 \\ &\quad + 2R_1^2 \sin \varphi dx_6 \wedge \epsilon_3 + 2R_2^2 \cos \varphi dx_5 \wedge \epsilon_3 \\ &\quad + 6 \tan \varphi d(f_2^{-1} k) \wedge dx_2 \wedge dx_3 \wedge dx_4. \end{aligned} \tag{9}$$

3 Supersymmetry of the solutions

3.1 Killing spinor equations of supergravity

The solutions we have considered in the last section are all supersymmetric, in particular they preserve 1/2 or 1/4 of the total 32 supersymmetries. The preserved supersymmetry can be studied using the supersymmetry transformations of the bosonic fields of the supergravity theory. For IIB string we may consider the supersymmetry variations of the dilatino and the gravitino, which are written as follows, in Einstein frame,

$$\begin{aligned}
\delta\lambda &= \frac{-1}{2\tau_2} \left(\frac{\tau^* - i}{\tau + i} \right) \Gamma^M \partial_M \tau (\eta_1 - i\eta_2) - \frac{i}{24} \Gamma^{MNP} G_{MNP} (\eta_1 + i\eta_2), \\
\delta\psi_M &= \partial_M (\eta_1 + i\eta_2) + \frac{1}{4} \omega_M^{ab} \Gamma^{ab} (\eta_1 + i\eta_2) + \frac{1}{8\tau_2} \left[\left(\frac{\tau - i}{\tau^* - i} \right) + \text{c.c.} \right] (\eta_1 + i\eta_2) \\
&\quad + \frac{i}{480} \Gamma^{M_1 \dots M_5} \Gamma_M F_{M_1 \dots M_5} (\eta_1 + i\eta_2) \\
&\quad - \frac{i}{96} (\Gamma_M^{NPQ} G_{NPQ} - 9 \Gamma^{NP} G_{MNP}) (\eta_2 + i\eta_1),
\end{aligned} \tag{10}$$

where

$$\tau = \tau_1 + i\tau_2 = \tau_1 + ie^{-\phi}, \tag{11}$$

$$G_{MNP} = i\sqrt{\tau_2} \frac{|1 - i\tau|}{\tau_2(1 - i\tau)} (F^{\text{RR}} - \tau F^{\text{NS}})_{MNP}. \tag{12}$$

Substituting the supergravity solutions for IIB Dp-branes we obtain the BPS conditions,

$$(\sigma_3)^{\frac{p-3}{2}} i\sigma_2 \otimes \bar{\Gamma}_{01\dots p} \epsilon = \epsilon, \tag{13}$$

with

$$\epsilon = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix},$$

where $\eta_{1,2}$ are two left-handed Majorana-Weyl spinors. $\bar{\Gamma}_M$ represent flat space gamma matrices. It is useful to remember that for F-strings the supersymmetry projection is $\sigma_3 \bar{\Gamma}_{01} \epsilon = \epsilon$.

For a D3-brane with B -field, the Killing spinor equation of the solution eq.(1) gives

$$h^{1/2} (i\sigma_2 \bar{\Gamma}_{0123} \cos \varphi - f^{-1/2} \sigma_1 \bar{\Gamma}_{01} \sin \varphi) \epsilon = \epsilon. \tag{14}$$

$\varphi \rightarrow 0$ is the limit of ordinary D3-brane while $\varphi \rightarrow \pi/2$ is relevant to the low energy noncommutative Yang-Mills theory as can be seen from eq.(2). We notice that the supersymmetry projection rule is the same as a D1-brane along 1-direction in that case.

For a D4-brane with $B_{34} \neq 0$, which can be obtained from eq.(1) via T-duality, similarly we have

$$h^{1/2}(\Gamma_{11}\bar{\Gamma}_{01234}\cos\varphi - f^{-1/2}\bar{\Gamma}_{012}\sin\varphi)\epsilon = \epsilon, \quad (15)$$

where ϵ for IIA theory is a Weyl spinor in 10D. Again at $\varphi = 0$ it is the same as ordinary D4-brane, while at $\varphi = \pi/2$ the projection condition becomes that of a D2-brane. along 12-directions.

Now let us consider intersecting cases. For intersecting pairs we have a system of BPS conditions, $(\Gamma_{(1)} - 1)\epsilon = (\Gamma_{(2)} - 1)\epsilon = 0$, with $\Gamma_{(1)}^2 = \Gamma_{(2)}^2 = 1$, $[\Gamma_{(1)}, \Gamma_{(2)}] = 0$, thus preserving 1/4 of the supersymmetries. If we substitute the solution of a noncommutative D4-brane pair intersecting over a 2d plane, eq.(4), into the supersymmetry transformation rule of IIA supergravity, we get the following set of conditions,

$$\begin{aligned} \Gamma_{(1)} &= h^{1/2}(\cos\varphi\Gamma_{11}\bar{\Gamma}_{01234} - f_1^{-1/2}f_2^{-1/2}\sin\varphi\bar{\Gamma}_{012}), \\ \Gamma_{(2)} &= h^{1/2}(\cos\varphi\Gamma_{11}\bar{\Gamma}_{03456} - f_1^{-1/2}f_2^{-1/2}\sin\varphi\bar{\Gamma}_{056}). \end{aligned} \quad (16)$$

For a noncommutative D3-brane pair solution eq.(6), sharing a 2d-plane, we have

$$\begin{aligned} \Gamma_{(1)} &= k^{1/2}(\sin\varphi f_2^{-1/2}i\sigma_2\bar{\Gamma}_{0123} + \cos\varphi f_1^{-1/2}\sigma_1\bar{\Gamma}_{01}), \\ \Gamma_{(2)} &= k^{1/2}(\cos\varphi f_1^{-1/2}i\sigma_2\bar{\Gamma}_{0234} - \sin\varphi f_2^{-1/2}\sigma_1\bar{\Gamma}_{04}). \end{aligned} \quad (17)$$

3.2 κ -symmetry and D-brane probes

Alternatively one can use κ -symmetry of D-brane action to study the supersymmetry of intersecting D-brane configurations. κ -symmetry is a fermionic gauge symmetry on the worldvolume, which produces a global worldvolume supersymmetry when combined with the global target space supersymmetry upon gauge fixing. Thus κ -symmetry plays an essential role in formulating supersymmetric D-brane actions [12]. And it is also useful in studying supersymmetrically intersecting configuration of D-branes [13, 14, 15]. For a brane probe configurations the fraction of preserved supersymmetry is determined by the following equation combined with the supersymmetry breaking condition of the gravity background,

$$(1 - \Gamma_\kappa)\epsilon = 0, \quad (18)$$

where ϵ is the spacetime supersymmetry parameter, and Γ_κ is an Hermitian traceless matrix, satisfying

$$\text{tr}\Gamma_\kappa = 0, \quad \Gamma_\kappa^2 = 1. \quad (19)$$

Γ_κ is nonlinear in $\mathcal{F} = F - B$, where F is the Born-Infeld 2-form field strength and B is the pull-back of the NS-NS two-form gauge potential. The explicit form will be important for our analysis.

$$\Gamma_\kappa = \frac{\sqrt{|g|}}{\sqrt{|g + \mathcal{F}|}} \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{\mu_1 \mu_1 \dots \mu_n \nu_n} \mathcal{F}_{\mu_1 \nu_1} \dots \mathcal{F}_{\mu_n \nu_n} J_{(p)}^{(n)}, \quad (20)$$

where g the induced metric by the map X , and

$$J_{(p)}^{(n)} = \begin{cases} (\Gamma_{11})^{n+\frac{p-2}{2}} \Gamma_{(0)} & \text{IIA,} \\ (-1)^n (\sigma_3)^{n+\frac{p-3}{2}} i\sigma_2 \otimes \Gamma_{(0)} & \text{IIB,} \end{cases} \quad (21)$$

and

$$\Gamma_{(0)} = \frac{1}{(p+1)! \sqrt{|g|}} \epsilon^{i_1 \dots i_{(p+1)}} \gamma_{\mu_1 \dots \mu_{(p+1)}}. \quad (22)$$

The matrix $\gamma_{\mu_1 \dots \mu_{(p+1)}}$ is the antisymmetrized product of the worldvolume gamma matrices γ_μ ,

$$\gamma_\mu = \partial_\mu X^M \Gamma_M, \quad (23)$$

where Γ_M are the spacetime gamma matrices.

Let us first consider Γ_κ for ordinary D-brane backgrounds. Naturally a Dp -brane probe embedded parallel to the background Dp -branes is supersymmetric, which means the supersymmetry condition from κ -symmetry consideration is identical to the one from Killing spinor equation of supergravity. For a noncommutative D-brane background we note that the value of B in the supergravity solution eq.(1) is set in the way that a probe D3-brane *without* U(1) Born-Infeld field becomes supersymmetric. For example we consider a D3-brane in the background of eq.(1), with worldvolume coordinates $(t, \xi^i), i = 1, 2, 3$,

$$X^0 = t, \quad X^i = \xi^i,$$

then we get, for $\mathcal{F} = -B$ i.e. $F = 0$,

$$\Gamma_\kappa = h^{1/2} (i\sigma_2 \bar{\Gamma}_{0123} \cos \varphi - f^{-1/2} \sigma_1 \bar{\Gamma}_{01} \sin \varphi),$$

which is exactly the same as the BPS condition eq.(14) obtained from supergravity. This is true for noncommutative Dp -brane pairs intersecting over $(p-2)$ d space. For example there are two commuting projection operators in eq.(16), each coinciding with Γ_κ 's for D4-brane lying on 1234-directions and 3456-directions, both with $F = 0$.

The situation is different with the supergravity solution of a noncommutative D3 pair intersecting over a plane, eq.(6). The value of B is shifted to give a D3 probe extended in 234-directions without Born-Infeld field supersymmetric. For the other

noncommutative D3-brane extended along 123-directions, the first line of eq.(17) corresponds to the Γ_κ of a D3-brane probe with $\mathcal{F}_{23} = \cot 2\varphi f_1^{-1}k$, or $F_{23} = 2 \csc 2\varphi$. In short, eq.(6) describes a supersymmetric D3-brane pair intersecting at 23-plane, with different values of \mathcal{F}_{23} , the difference being $2 \csc 2\varphi$.

4 Baryon for noncommutative AdS/CFT

It was first suggested in [16] that a D5-brane can be used as the source of fundamental strings whose end points on D3-branes are considered as baryon in the context of AdS/CFT correspondence, and the baryon mass was calculated following this idea in [17]. The fundamental strings in turn can be studied in terms of electrically charged solitons of D5-brane Born-Infeld action. The supersymmetry condition for the baryonic D5-brane was first obtained in [18] and the explicit solution was found and analyzed in detail in [19]. M-theory generalization was also achieved in [20].

N coincident D3-branes at the origin generate the following supergravity solution,

$$\begin{aligned} ds^2 &= f^{-1/2} ds^2(E^{1,3}) + f^{1/2} (dr^2 + r^2 d\Omega_5^2), \\ G_{(5)} &= 4R^4 (\omega_{(5)} + \star \omega_{(5)}), \end{aligned} \quad (24)$$

which is $\varphi = 0$ case of eq.(1), and we use the near horizon limit $f = R^4/r^4$. The parameter R is given by $R^4 = 4\pi gN$. We will study the behaviour of a D5-brane probe with unit tension, wrapping S^5 part of the above background. The Hanany-Witten effect [21] is realized in terms of the Wess-Zumino term in the D-brane action. The configuration is represented as follows,

$$\begin{array}{llllllllll} D3 : & 1 & 2 & 3 & - & - & - & - & - & \text{background} \\ D5 : & - & - & - & 4 & 5 & 6 & 7 & 8 & - \text{probe} \\ F1 : & - & - & - & - & - & - & - & 9 & \text{soliton} \end{array} \quad (25)$$

where 4,5,6,7,8-directions are 5 angles of S^5 , 9-direction is the radial direction. The D3-brane supergravity background gives the projection rule

$$i\sigma_2 \bar{\Gamma}_{0123} \epsilon = \epsilon, \quad (26)$$

where ϵ is a covariantly constant spinor of S^5 . For our discussion in this section it is sufficient to consider bosonic sector of D-brane action,

$$S = - \int d^6 \xi \left\{ e^{-\phi} \sqrt{-\det(g + \mathcal{F})} - V \wedge G_{(5)} \right\}. \quad (27)$$

Let $\sigma^\mu = (t, \theta^i), i = 1, \dots, 5$ be the worldvolume coordinates of D5-brane and we choose the static gauge to fix the worldvolume diffeomorphisms

$$X^0 = t, \quad X^{i+3} = \theta^i \quad (i = 1, 2 \dots 5). \quad (28)$$

And we take $X^1 = X^2 = X^3 = 0$, the only activated scalar $X^9 = r$. From the equation $(1 - \Gamma_\kappa)\epsilon = 0$ and the consistency with the background eq.(26) it can be derived that the BPS condition is [20]

$$F_{0i} = \partial_i(r \cos \theta), \quad (29)$$

where θ is the polar angle of S^5 . With this BPS equation and ansatz of $SO(5)$ symmetry, the Gauss law leads to

$$\left[\sin^4 \theta \frac{(r \cos \theta)'}{(r \sin \theta)'} \right]' = -4 \sin^4 \theta, \quad (30)$$

which can be solved analytically [19]

$$r(\theta) = \frac{A}{\sin \theta} \left[\frac{\eta(\theta)}{\pi(1 - \nu)} \right]^{1/3}, \quad \eta(\theta) = \theta - \pi\nu - \sin \theta \cos \theta, \quad (31)$$

where A is an arbitrary scale factor reflecting the conformal symmetry and ν is an integration constant, which is related to the number of fundamental strings connecting the D3 and D5-brane.

Now let us turn to the noncommutative case. The supersymmetry projection rule of the noncommutative D3-brane background is obtained already. Because of the term representing D1-brane charge it is obvious that trivial D5-brane configuration is not supersymmetric in this background. It is still true if we allow for arbitrary Born-Infeld and scalar excitations on the D5-brane. A hint is given from the example studied earlier, a noncommutative D3-brane pair intersecting over 2-plane. Let us consider a D7-brane, with nonzero \mathcal{F}_{23} . Using the supergravity background eq.(1) and expanding eq.(20), we get

$$\Gamma_\kappa = \frac{1}{\sqrt{h^2 + f(\mathcal{F}_{23})^2}} (h \bar{\Gamma}_{02345678} i \sigma_2 + f^{1/2} \mathcal{F}_{23} \bar{\Gamma}_{045678} \sigma_1), \quad (32)$$

where $x_{4,5,6,7,8}$ are angular coordinates of S^5 . It is straightforward to check that Γ_κ in eq.(14) and eq.(32) commute with each other, when

$$\begin{aligned} \mathcal{F}_{23} &= h \cot \varphi \\ &= 2 \csc 2\varphi - B_{23}. \end{aligned} \quad (33)$$

So in the supergravity background of noncommutative D3-branes, a D7-brane along 2345678-directions is supersymmetric when a constant Born-Infeld gauge field $2 \csc 2\varphi$ is turned on, in addition to the NS-NS background.

We are thus led to consider the Born-Infeld action of D7-brane, using the induced metric in the background of eq.(1). It is

$$\begin{aligned} S = & -T_7 \int d^8 \xi e^{-\phi} \sqrt{-\det(g + \mathcal{F})} + T_7 \int d^8 \xi A_\alpha \partial_\beta X^{M_1} \dots \partial_\gamma X^{M_7} F_{M_1 \dots M_7} \\ & + T_7 \int d^8 \xi A_\alpha \mathcal{F}_{23} \partial_\beta X^{M_1} \dots \partial_\gamma X^{M_5} F_{M_1 \dots M_5}, \end{aligned} \quad (34)$$

where $F_{M_1 \dots M_7}$ is the dual of RR 3-form field strength in eq.(1), which couples to D1-brane, while $F_{M_1 \dots M_5}$ is the RR 5-form field strength which couples to D3-brane and is self-dual.

Following [19] we can proceed in the same way to write the action in detail in the near horizon limit, for D7-brane with SO(5) symmetry. D7 worldvolume coordinates are $\xi^\mu = (t, y, z, \theta^i)$, $i = 1, 2, \dots, 5$

$$X^0 = t, \quad X^2 = y, \quad X^3 = z, \quad X^{i+3} = \theta^i, \quad (35)$$

and $X^1 = 0$ and the radial coordinate as the only activated scalar, $X^9 = r$ as function of $y, z, \theta \equiv \theta^1$.

$$S = \frac{R^4 T_7 \Omega_4}{\sin \varphi} \int dt dy dz d\theta \sin^4 \theta (-\sqrt{r^2 + r_\theta^2 - F_{0\theta}^2 + K \sin^2 \varphi + 4A_0}), \quad (36)$$

where

$$\begin{aligned} K = & (r_y^2 + r_z^2)(r^2 - F_{0\theta}^2) + 2F_{0\theta} r_\theta (F_{0y} r_y + F_{0z} r_z) - (r^2 + r_\theta^2)(F_{0y}^2 + F_{0z}^2) \\ & - r^2 f h^{-1} (F_{0z} r_y - F_{0y} r_z)^2, \end{aligned} \quad (37)$$

and $r_y = \partial_y r$, etc.

Without help from BPS conditions solving the equations from above action should be very complicated. We can find the BPS equations from the condition $(1 - \Gamma_\kappa)\epsilon = 0$, with Born-Infeld gauge potential A_0 and scalar field r turned on to describe fundamental strings attached on D7, coming from the noncommutative D3-branes lying at the horizon. And it has to satisfy the condition $[\Gamma_{\text{sugra}}, \Gamma_\kappa] = 0$. It turns out that the BPS condition is again the same form,

$$F_{0i} = \partial_i(r \cos \theta). \quad (38)$$

the only difference being now i includes y, z . K vanishes when we use this BPS equation, so the equation of motion is simplified substantially. Employing the decoupling limit eq.(2), we finally get

$$\sin^{-4} \theta \partial_\theta \left(\sin^4 \theta \frac{\partial_\theta(u \cos \theta)}{\partial_\theta(u \sin \theta)} \right) + \frac{\Theta^2}{2} (\partial_y^2 + \partial_z^2) u^2 = -4, \quad (39)$$

where \bar{y}, \bar{z} represent the coordinates which are noncommutative in the dual large N Yang-Mills theory satisfying $[\bar{y}, \bar{z}] \sim \Theta$. At $\Theta = 0$ we reproduce eq.(30) but for $\Theta \neq 0$ this is a nonlinear partial differential equation, and the usual technique of separation of variables is not useful in this case. The dependence of u on θ encodes the flavor SU(4) structure of the baryon, while the dependence on \bar{y}, \bar{z} should describe the size of baryon in the noncommutative plane at a specific energy scale u . It is easily seen that if $u(\theta, \bar{y}, \bar{z}; \Theta)$ is a solution to the equation, $u(\theta, k\bar{y}, k\bar{z}; \Theta/k)$ is also a solution, which implies the size of the baryon in $\bar{y}\bar{z}$ -plane scales as Θu . This is consistent with the philosophy of noncommutative field theory introducing Θ as a parameter of nonlocality.

5 Discussion

In this paper we have studied intersecting noncommutative p -branes which are BPS. The lesson is that there exist orthogonally intersecting D-brane pairs which are supersymmetric, with the number of total transverse directions 2 mod 4, as well as 0 mod 4. The usual harmonic superposition rule cannot be applied directly to get the supergravity solutions, but we can easily obtain the solutions using tilted D-branes via T-duality. Here we considered solutions with only one component of B -field which is pure magnetic, but we can also consider B -fields with larger ranks, and $B_{0i} \neq 0$ cases, which generate electric \mathcal{F} . The gauge bundle on the D-brane worldvolume then represent a bound state of D-brane and fundamental strings. Especially supergravity solutions of D3-branes with self-dual B-fields are obtained in [22]. Supersymmetric configurations of D-branes with nonzero F in supergravity backgrounds with B are analyzed systematically using κ -symmetry and applied to study the nonlinear deformations of the instanton equations in [23].

For baryonic branes, it will be interesting to extend this study to M-theory, following [20]. Because of the self-duality of C -field on M5-brane worldvolume, to get supersymmetric configuration we need to consider M9-branes. There are suggestions on M9-brane action [24] as a supersymmetric nonlinear sigma model. The lack of κ -symmetry thereof makes it difficult to find the supersymmetric configuration. We leave the subject of solving the equation eq.(39) and investigating baryonic M9-brane for future work.

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